Vector Addition (Mathematical)

There are three possible situations when combining vectors:

I. Parallel Vectors

These include vectors that have the same or opposite directions.

To add these:

1. Convert to a $+/-$ direction system.
2. Add the magnitudes using regular addition.
3. Convert the result back to the original direction system.

Example 1

$\vec{v}_1 = 250\ m/s\ [E]$, $\vec{v}_2 = 100\ m/s\ [W]$. Find $\vec{v}_1 + \vec{v}_2$.

II. Perpendicular Vectors

These include vectors whose directions are perpendicular.

To add these:

1. Sketch a right-triangle consisting of the two vectors being added and the resultant.
2. Find the magnitude of the resultant using the Pythagorean relation.
3. Find the direction of the resultant using the basic trigonometric functions.

Example 2

$\vec{a}_1 = 10\ m/s^2\ [N]$, $\vec{a}_2 = 15\ m/s^2\ [W]$. Find $\vec{a}_1 + \vec{a}_2$.

III. Vectors at any Angle

These include vectors that do not fall into the first two categories. There are two different methods that may be used to add these vectors.

Method I

To add two vectors:

1. Resolve each vector into its horizontal and vertical components.
2. Add up the vertical components.
3. Add up the horizontal components.
4. Add the resulting vertical and horizontal components (as perpendicular vectors) to obtain the final resultant.

Example 3

$\vec{F}_1 = 12.0\ N\ [10°\ N\ of\ E]$, $\vec{F}_2 = 8.0\ N\ [30°\ W\ of\ N]$. Find $\vec{F}_1 + \vec{F}_2$.

Method II

To add two vectors:

1. Sketch a triangle (non-right angled) consisting of the two vectors being added and the resultant.
2. Use the Law of Cosines and the Law of Sines to find the resultant.

Example 4

$\overrightarrow{F_1} = 12.0\ N\ [10°\ N\ of\ E]$, $\overrightarrow{F_2} = 8.0\ N\ [30°\ W\ of\ N]$. Find $\overrightarrow{F_1} + \overrightarrow{F_2}$.

Example 5

$\vec{v_1} = 100\ m/s\ [20°\ W\ of\ S]$, $\vec{v_2} = 200\ m/s\ [30°\ S\ of\ W]$. Find $\vec{v_1} + \vec{v_2}$.

Homework

Vectors Worksheet #2

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Solve each of the following vector problems using mathematical methods only.

1. A boat sails in a straight line 20 *km* $\left[30° \; E \; of \; N\right]$. What are the components of its displacement to the north and east? (17 *km* [N], 10 *km* [E])

2. A cannon fires a cannonball with a speed of 100 *m/s* at an angle of $20°$ above the horizontal. What are the horizontal and vertical components of the initial velocity of the cannonball? (94 *m/s* [*H*], 34 *m/s* [*V*])

3. One car travels east at 100 *km/h* and another travels north at 100 *km/h*. Are their velocities equal? Are their speeds equal?

4. You walk 30 *m* south and 30 *m* east. Find the magnitude and direction of the resultant displacement. (42 *m* $\left[SE\right]$)

5. A student walking his pet dog walks 0.4 *km* [N], then 0.3 *km* [E], and then returns home. The whole walk took 0.5 hours.
   a. What was the total displacement for the whole walk? (0)
   b. Find the total displacement for the first two legs of the walk. (0.5 *km* $\left[37° \; E \; of \; N\right]$)
   c. What was the displacement of the final part of the walk home? (0.5 *km* $\left[53° \; S \; of \; W\right]$)
   d. What was the average speed for the whole walk? (2.4 *km/h*)
   e. What was the average velocity for the first two parts of the walk? (1.0 *km/h* $\left[37° \; E \; of \; N\right]$)

6. A train moving at a constant speed of 100 *km/h* travels east for 40 min, then $30°$ east of north for 20 min, and finally west for 30 min. What is the train's average velocity for the trip? (30 *km/h* $\left[40° \; N \; of \; E\right]$)

7. A man walks 600 *m* $\left[47° \; N \; of \; E\right]$, then 500 *m* $\left[38° \; W \; of \; N\right]$, then 300 *m* $\left[29° \; S \; of \; W\right]$, and finally 400 *m* $\left[13° \; E \; of \; S\right]$. Find his resultant displacement. (306 *m* $\left[13° \; W \; of \; N\right]$)

8. For the following path of 50 *m* $\left[47° \; W \; of \; N\right]$, 22 *m* $\left[43° \; N \; of \; W\right]$, 30 *m* $\left[60° \; S \; of \; E\right]$, 30 *m* $\left[E\right]$, and 44 *m* $\left[75° \; E \; of \; N\right]$, find
   a. the total distance traveled. (176 *m*)
   b. the total displacement. (49 *m* $\left[NE\right]$)
   c. the direction of the most direct route back to the start. ($\left[SW\right]$)